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Ecological inference under unfavorable conditions: Straight and split-ticket voting in diverse settings and small samples

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ABSTRACT

Problems of ecological inference have long troubled political scientists. Thomsen's (1987) estimator for ecological inference has been shown to produce estimates close to the individual level estimates for transitions across elections, but it is unknown how well it performs under unfavorable conditions. We fill this void by testing the estimator as the across-unit variance increases and introduce a new procedure to examine the bias of the estimates as the number of aggregate units decreases. Looking at partisan voting patterns across races within the 2000 general election in Florida counties and taking advantage of ballot image data to study straight-ticket voting we demonstrate that the estimator performs well in both heterogeneous societies and when the number of aggregate units is limited.

1. Introduction

Ecological inference problems are well-known in the field of political science. Essentially, scholars face a problem of deficient data, as often they must make assertions about some micro-level relationship through the employment of aggregate-level data. Researchers' concerns about the cumbersomeness of the assumptions required to generate reliable estimates of the lower-level phenomenon have caused them to raise questions about the quality of results procured through such analyses and also stimulated their design of a number of estimation strategies.

Researchers commonly use ecological inference techniques to determine voter transition rates across elections, i.e., the proportion of voters who vote for the same party across two elections and the proportion who switch to another political party. For such analyses, the Thomsen (1987) estimator exhibits certain characteristics that permit it to generate estimates from aggregate data that are close to the individual-level estimates (Thomsen et al., 1991; Achen, 2000; Hanmer and Traugott, 2004; Park, 2008a). Moreover, Park (2008a) shows that it performs better than other common estimators. Despite its demonstrated success, we possess limited knowledge of how the estimator performs under less than ideal conditions. Concerns about an estimator's performance when substantial diversity exists within or across the units of analysis, as well as when the sample size is small, are common to all estimators. Thomsen (1987, 55), however, makes an important identification assumption about the relationship between the individual-level units and how they relate to the aggregated units, namely that "the variation between individuals has the same structure as the variation between districts." Since ecological inference is essentially a problem of aggregation, examining circumstances when this sort of homogeneity is not present across the aggregate

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units is highly important. Unfortunately, researchers have scant information to bring to bear when attempting to judge the performance of the Thomsen estimator in these situations.

We fill these voids in the literature, testing the estimator as both the across-unit variance increases and the number of aggregate units decreases. We do so by examining voting patterns within a single election (straight and split-ticket voting) in ten individually and collectively diverse Florida counties in the 2000 general election. These data are particularly well suited to test the Thomsen estimator. Using ballot image data as our measure of actual vote proportions, we find that the Thomsen estimator is robust to individually and collectively diverse settings. In addition, the introduction of a new resampling procedure reveals the stability of the estimates in the context of limited numbers of aggregation units, suggesting that thirty to fifty such units are more than sufficient to determine the underlying individual-level relationship. This bodes well for scholars of comparative and U.S. state and local politics, who often use data sets fixed at this size (e.g., fifty states). Studies of electoral dynamics frequently turn to ecological inference techniques to infer individual-level relationships from aggregate-level electoral results in the United States and, among others, Czechoslovakia, India, Italy, Japan, Post-Soviet Russia, Uruguay, and Western Europe (e.g., Alexseev, 2006; Altman, 2002; Benoit et al., 2006; Burden, 2009; Burden and Kimball, 1998, 2002; Chandra, 2009; Golder, 2003; Hanmer and Traugott, 2004; Kopstein and Wittenberg, 2009; Hanmer et al. 2010). The confidence in the Thomsen estimator created by our results suggests a number of potential applications that can be implemented with easy to use software that we have provided at http://cpc.snu.ac.kr/computing/stata.

2. Ecological inference and the Thomsen estimator

Ecological inference is essentially a problem of statistical under-identification. Researchers have interest in the process behind some micro-level occurrence, but the aggregate data available are insufficient for such a determination. To draw inferences in such cases, researchers must make strong assumptions upon which both the validity and accuracy of the estimator heavily rely. While collecting individual-level data would present a more straight-forward manner of addressing the relationship in question, the cost or availability of such data sets often leaves scholars with no choice but to use ecological inference techniques. This realization has spurred significant discussion and advances in the field (Achen and Shivley, 1995; Adolph and King, 2003; Adolph et al., 2003; Brown and Payne, 1986; Calvo and Escolar, 2003; Cleave et al., 1995; Elff et al., 2008; Greiner and Quinn, 2009; Herron and Shotts, 2003a,b, 2004; Johnston and Pattie, 2000; King, 1997; Rosen et al., 2001; Tam Cho, 1998; Tam Cho and Gaines, 2004; Thomsen, 1987; Wakefield, 2004), with a number of statistical techniques suggested to address the problem of ecological inference.

One common application of these techniques is to voter transition rates across elections. The attempt to estimate these rates from aggregate data presents a prime example of a typical ecological inference problem. Scholars often wish to estimate partisan loyalty or defection rates of voters in two consecutive elections (the transitions), but may lack adequate information to do so. Usually, the collection of individual-level data to determine these rates is either impossible or unfeasible, especially with regards to questions of a historical nature or when survey data do not exist (as is often the case in non-Western countries). When survey data do exist, they often cannot be reduced to smaller aggregate units (such as a national survey sample into states or congressional districts) because the samples within each unit are too small for reliable analysis. In addition, it is well documented that inaccurate reporting of voting behavior can plague survey data (see, e.g. Belli et al., 2001; Duff et al., 2007).

In such circumstances, aggregate-level records are usually more accurate, extensive, and collected at relatively small geographic units. Aggregate-level data, however, provide us with a problem of unknown data. While the researcher knows the electoral support received by n competing parties for two consecutive elections (marginal probability) in each observational unit, no information exists about the individual counts for all possible n^2 cells (joint probabilities). In this instance, the goal of ecological inference is to infer the unknown individual voting choice based on the known aggregate information (marginal vote fractions).

While researchers have employed a number of ecological inference techniques to analyze voter transition rates (see Altman, 2002; Benoit et al., 2006; Brown and Payne, 1986; Burden, 2009; Burden and Kimball, 1998, 2002; Chandra, 2009), Thomsen's (1987) estimator is particularly well suited to do so. The intuition behind the Thomsen estimator is that two consecutive elections should be treated symmetrically (as opposed to treating the second election as the dependent variable influenced by the first election), with the two election outcomes being the result of one common latent factor. Since this latent factor drives a voter's choices in the two elections and links voting behavior across elections, we may conceptualize it as an underlying partisanship dimension.

Call this latent variable at the individual level d_i^*, and the vote choice in a given election at time t as d_t, which would be a binary variable. Define $\Phi(\bullet)$ as the cumulative distribution function of the standard normal distribution. In its simplest form, we may write.

$$\mathrm{Prob}(d_{ti}=1)=\Phi\left(\alpha_t+\beta_t d_i^*+e_{ti}\right). \tag{1}$$

When this is aggregated to the district level, the aggregate outcome D_{tj} we observe in a given district j should correspond to the expected vote fractions in the district using the above equation. Assuming with Thomsen that the underlying dimension d_{ij}^* is normally distributed with mean D_j^* and a constant variance σ^2, we find the average of both sides of the above equation:

$$E\left(d_{tij}\right)=D_{tj}=\int_{-\infty}^{\infty}\Phi\left(\alpha_t+\beta_t d_{ij}^*+e_{tij}\right)\phi\left(d_{ij}^*\middle|D_j^*,\sigma^2\right)\partial d_{ij}^*. \tag{2}$$

By carrying out the integral, we obtain (Achen and Shivley, 1995, p. 184):

$$E(d_{tij}) = D_{tj} = \Phi\left(\frac{\alpha_t + \beta_t D_j^*}{\sqrt{1+\beta_t^2\sigma^2}}\right). \tag{3}$$

Or more simply.

$$D_{1j} = \Phi\left(A_1 + B_1 D_j^*\right) \text{ and } D_{2j} = \Phi\left(A_2 + B_2 D_j^*\right). \tag{4}$$

Since the D_j^* term here is latent which we do not observe, we want to factor out the term by writing:

$$\Phi^{-1}(D_{1j}) = A_0 + B_0\Phi^{-1}(D_{2j}). \tag{5}$$

Of course, the model is not identified and will not allow us to estimate the parameters that involve the latent term, but it at least establishes that the inverse-probit of the two aggregate electoral outcomes are linearly related and joint-normally distributed. With this result, we can identify the voter transition rates.[1]

In addition to this micro-foundation modeling now incorporated into the estimator (Achen, 2000), the Thomsen estimator maintains the advantage of being able to deal with the non-linearity that arises with voter transition rates, the problems of which can be more severe than in usual aggregate data sets (Park, 2008a). Failing to deal with the problem of nonlinearity native to the voter transition model causes bias and inconsistency. The Thomsen model is non-linear in its specification, which guards against these concerns and prevents the estimates of voting probabilities from falling outside of the logical range (0–1), a problem with other methods such as ecological regression (Goodman, 1953, 1959). The estimator has been demonstrated to provide favorable (Cleave et al., 1995) or superior estimates to other existing strategies, such as King's (1997) ecological inference method (Park, 2008a). Though the comparison of estimators is not our objective here, our analyses confirm that the Thomsen estimator consistently outperforms King's EI in estimating the outcomes of interest here (see more below, including Appendix Tables A and B, for a comparison of these estimates).[2]

The estimator has been shown to produce estimates very close to the individual voter transition rates across elections in several contexts. In his review of the estimator, Achen (2000) concludes that tests of the estimator suggest it to be quite successful. Thomsen et al. (1991) and Thomsen (2000) show that the estimator performs well using aggregate data for elections in Denmark, Finland, and Sweden over four decades. Park (2008a) successfully applies this estimator to both British parliamentary elections in the 1960s and South Korean presidential elections in the 1990s. The Thomsen estimator has also been used by Park (2008b) to generate reliable estimates of voter transition rates for different groups in South Korea in presidential elections in the 1980s. Hanmer and Traugott (2004) generate remarkably close estimates of individual-level ballot image data for partisan voting patterns from one race to another for the 2000 general election in Oregon.

While researchers largely employ the Thomsen estimator to estimate voting patterns across elections, it can also be used to estimate patterns within an election (i.e., from one race to another within the same ballot). The estimator's success in this arena opens the door to a number of applications. The extent to which (and the reasons why) voters split their ballots is a central question in the voting behavior literature. Over the past two decades, scholars have expended substantial effort to comprehend better the circumstances under which split-ticket voting and divided government occur (Alesina and Rosenthal, 1995; Beck et al., 1992; Burden and Kimball, 1998; Fiorina, 1996; Jacobson, 1990; Lewis-Beck and Nadeau, 2004; Roscoe, 2003). Conventional wisdom related to the potential increased party polarization over the past two decades (Abramowitz and Saunders, 1998; Carsey and Layman, 2006; Jacobson, 2000; Layman and Carsey, 2002) suggests that the incidences of split-ticket voting and divided government should decline significantly, which raises interesting and important questions about why either might continue to prevail at such high rates. Some, most notably Fiorina (1996), assert that the answer to these questions derives mainly from the general population's interest in balancing the power of the two parties, which serves as a manner to moderate public policy that better matches their less extreme preferences. Accurate estimates of how many voters split their ballots (and why) can thus help scholars understand the extent to which divided government (and policy-balancing) is actually desired by the population (Burden and Kimball, 1998). Additionally, given the interest in partisan polarization, as well as rapid changes in election procedures (such as early voting, absentee voting, and the introduction of electronic voting machines) that might alter partisan behavior within a single election, techniques to provide reliable estimates of individual-level behavior from aggregate data (such as straight and split-ticket voting) are becoming increasingly important.

Previous investigations have attempted to study straight and split-ticket voting using other ecological inference estimators with mixed success (Burden and Kimball, 1998), but the Thomsen estimator, with its partisan micro-foundation, is particularly well suited for studying partisan voting patterns across races within a single election. This is especially true when thinking about the assumed *common* latent partisanship dimension: in a given election where the

[1] An additional challenge in estimating the voter transition rates, if not more difficult than the ecological inference process itself, is the problem of dealing with multiple parties ("the R $\times$ C case"). Here, we used the iterative proportional fitting (IPF) process after estimating all the possible bivariate combinations of voter transitions. IPF, also known as "raking," is a mathematical scaling procedure to adjust a matrix of any dimension to converge to some pre-defined row and column totals, where the constraining row and column totals are obtained from alternative sources, and has been used to adjust the cell frequencies or proportions from survey estimates to conform to the marginal distribution of the same census variables (Deming and Stephan, 1940). The theoretical underpinnings of IPF blend well with the task of dealing with the R $\times$ C case in the ecological inference context and we use it here (see Park, 2008a for more details).

[2] EI analysis is run in R using the ecological inference code integrated into *Zelig* (Imai et al., 2007a,b), which implements models using a nonlinear least squares approximation (Wittenberg et al., 2007). In contrast to earlier ecological inference approaches that are Bayesian in nature (see for example Rosen et al., 2001), this strategy implements a frequentist approximation of these Bayesian models. As such, it is not Bayesian by design and does not require priors or starting values to be specified.

same set of voters participates, the assumption will most likely hold. In two elections over time, partisanship may decay and the electorate may change.

We focus here on its performance under less favorable conditions than those where researchers have previously used this estimator. For example, we possess limited knowledge about the performance of the Thomsen estimator when applied to diverse aggregate units. The studies described above all generate estimates close to the individual-level parameters in the context of relatively homogenous settings, such as Oregon, Scandinavia, and South Korea (Thomsen et al., 1991; Thomsen, 2000; Hanmer and Traugott, 2004; Park, 2013). While the ability to do so is important for the study of comparable cases, we have no investigations from which to judge the estimates based on heterogeneous societies. This dearth of information is significant because Thomsen's identifying condition requires the ecological correlation in the aggregate electoral data to equal that of the individual correlations between the vote propensities, an assumption he maintains will hold as the aggregate units tend toward homogeneity in comparison to one another. Thomsen's (1987) homogeneity assumption is another expression for assuming away the “grouping problem” that is at the heart of aggregation bias (see especially page 52). For example, King (1997) notes that if the grouping process (that is, the aggregation of individuals in the same aggregate unit) is correlated with the dependent variable, the ecological estimates will be biased (see page 50). Thus, violations could have severe consequences for the estimator's performance.[3]

Although subsequent extensions relax and appease some concerns about this assumption (Achen, 2000), we lack empirical evidence to support this claim, as the studies cited above cannot inform us about the robustness of the estimator to heterogeneity both across and within the aggregated units of analysis. Or, to posit the concern in another way, we cannot determine from existing studies whether or not “potential applications are … restricted to homogenous settings with good herring on the menu” (Achen, 2000, 16).

Additionally, we have minimal knowledge about how well the Thomsen estimator performs with a limited number of aggregate units. In instances where the number of units is small, concerns emerge about the performance of any estimator. While the Thomsen estimator performs well under favorable circumstances, scholars have yet to investigate its ability to recover the individual relationships in situations with small samples. In most cases, the number of aggregate units is fixed (e.g., fifty states), leaving no ability to expand the number of units by, for example, simply collecting more data. Given the fixed nature of the data, to generate reliable results the estimator must be robust to changes in the sample size. As such, identifying the performance of the Thomsen estimator under unfavorable conditions is crucial to understanding the ultimate utility of its application in addressing substantive problems with aggregate data.

[3] Note that traditional regression approaches such as Hanushek et al. (1974) tackled this problem by controlling for a multitude of relevant factors, hoping to make the aggregation units as homogenous as possible after the control.

3. Data

We fill this void by first examining the performance of the Thomsen estimator in the setting of ten individually and collectively diverse counties in Florida in the context of the 2000 general election.[4] The data were provided by *The Washington Post* as part of the media's effort to investigate the issues with ballots and voting technology that arose during this election.[5] For the analyses below, we focus on four counties (Miami-Dade, Lee, Palm Beach, and Sarasota) that represent the substantial racial/ethnic and partisan diversity found across the locales.[6] Along racial and ethnic lines, they cover the range of the Blau (1977) diversity index for all ten counties (from 0.19 to 0.59)[7]; Sarasota and Miami-Dade are on the two extremes, with Lee (0.31) and Palm Beach (0.47) almost equidistant from the national average (0.40) in 2000.[8] In comparison to the national population size of African Americans (12.3%) and Latinos (12.5%), these index scores correspond to much smaller minority communities in Sarasota (4.2% and 4.3%) and Lee (6.6% and 9.5%), a roughly comparable breakdown in Palm Beach (13.8% and 12.4%), and a substantially larger minority population in Miami-Dade (20.3% and 57.3%).

With regards to partisan diversity (as measured by presidential vote choice in 2000), the four selected counties equally cover the range of analyzed counties (from 0.45 to 0.53), with Palm Beach being the second least diverse county, Lee and Miami-Dade in the middle, and Sarasota among those with the greatest partisan diversity. Substantial partisan variation also exists across the individual precincts of each county, as measured by the Democratic percent of the presidential vote. With a standard deviation in this vote choice ranging from 0.07 to 0.2 for the ten counties, Lee and Sarasota Counties (both 0.09) exhibit lower levels of precinct partisan heterogeneity, while Palm Beach County (0.13) resides in the middle of the distribution of counties and Miami-Dade possesses the greatest diversity across its precincts (0.2). This substantial variation in voting-relevant characteristics provides ample heterogeneity in which to examine the estimator when a key assumption is violated.

In contrast to many ecological inference analyses that rely on survey data to determine accuracy, we had the extraordinary opportunity to utilize as our measure of the “truth” ballot image data from every single ballot actually recorded in these counties in the 2000 election (almost 2.8

[4] These counties include Broward, Highlands, Hillsborough, Lee, Marion, Miami-Dade, Palm Beach, Pasco, Pinellas, and Sarasota.

[5] The American National Election Study also made these data available through their 2000 Florida Ballot Project (available: http://www.electionstudies.org/florida2000/data/data_files.htm).

[6] These counties are selected for illustrative purposes only. Results are similar for all counties and can be found in the Web Appendix.

[7] This index ranges from zero to one, with zero indicating no diversity (i.e., the entire population is white) and higher scores reflecting greater heterogeneity in the population makeup. It is calculated by subtracting from one the summed squares of each group's proportion of the total population. More formally: Diversity $= 1 - \sum_j p_j^2$.

[8] Both Palm Beach and Miami-Dade exhibit noticeably higher rates of racial/ethnic diversity than Multnomah, OR (0.36), a locale previously analyzed using the Thomsen estimator (Hanmer and Traugott, 2004).

Table 1A
Tabulated vote proportions from ballot images, Miami-Dade 2000.

		U.S. Senate race				
		Republican	Democrat	Others	Resid. votes	Total
President	Bush	35.55%	3.13%	1.02%	3.73%	265,211
	Gore	3.03%	42.23%	2.15%	3.66%	311,879
	Others	0.29%	0.50%	0.21%	0.10%	6691
	Resid. votes	0.73%	1.54%	0.44%	1.71%	26,927
	Total	241,804	289,396	23,328	56,180	610,708

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 2.72% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

Table 1B
Estimated vote proportions using Thomsen estimator from precinct-level data ($N = 613$), Miami-Dade 2000.

		U.S. Senate race			
		Republican	Democrat	Others	Resid. votes
President	Bush	35.02%	3.44%	0.56%	4.41%
	Gore	3.25%	41.43%	2.71%	3.68%
	Others	0.56%	0.46%	0.02%	0.05%
	Resid. votes	0.76%	2.06%	0.53%	1.06%

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 2.72% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

million ballots in total). Doing so allows us to avoid concerns with survey data about the lack of an adequate number of individual cases per aggregation unit, as well as errors in the responses. Because we know how each individual vote is recorded (as opposed to one's recall of how he/she voted), we can directly compare the Thomsen estimates to the actual vote proportions cast. To do so, we use these ballot images to examine the occurrence of straight and split-ticket voting for president and member of the U.S. Senate, the top two races on the ballot, and then aggregate the individual ballot images to the precinct-level to use as our aggregate unit of analysis.[9] Of course, one of the big lessons from the 2000 election was that voters make errors (Caltech/MIT, 2001; Herrnson et al., 2008; Wand et al., 2001). If anything, our use of these data provides a more stringent test, especially in the case of Palm Beach County where the butterfly ballot (see Wand et al., 2001), an exogenous shock, served to disrupt the underlying micro-foundation.[10]

4. Results

4.1. Recovering voting patterns in diverse settings

We examine the Thomsen estimates for Miami-Dade County first. As noted earlier, Miami-Dade is diverse relative to the other counties in Florida and the nation as a whole. This diversity is also marked by substantial heterogeneity in its Latino composition. Roughly half of the Latino population, for example, is of Cuban descent, and such individuals tend to be much more likely to vote Republican than the rest of the dominant Latino communities. In addition to the candidates for the two major parties, we examine other candidates (votes for Nader added with votes for all other candidates) and residual votes (combination of undervotes and overvotes).[11] Table 1A presents the actual vote proportions tabulated from ballot images of straight and split-ticket voting in the 2000 election, while Table 1B presents the estimated vote proportions from precinct-level data. As is evident, our Thomsen estimates from the aggregate-level data are remarkably similar to the actual vote proportions garnered from the individual-level ballot images. Only in the case of voting for Gore and the Democratic candidate for Senate, Bill Nelson, does the difference (0.80 points) in the estimated and actual vote proportions surpass three-quarters of a percentage point, and only in four of the other fifteen cells does the difference exceed half of a percentage point. Thus, scholars employing precinct-level data and the Thomsen estimator would conclude correctly, as evidenced by the ballot image data, that a slightly larger percentage of Bush voters cast a vote for Nelson than Gore voters cast a vote for the Republican Senate candidate, Bill McCollum.[12] The significant similarity between the actual and estimated vote proportions suggests that the Thomsen estimator produces reliable predictions when there is substantial demographic diversity within and across the units of aggregation.

In addition to generating results that are incredibly close to the actual vote proportions in the county, the Thomsen estimator also captures a number of key relationships between vote transitions that are missed by other estimators. That is, as well as the obvious value of getting closer to the actual results than other techniques, the Thomsen estimator ensures that the substantive conclusions supported by the analysis are consistent with the relationships found in the individual-level data. This contrasts sharply with another common estimator, King's EI, which does not recover several important patterns in the data and thus would lead researchers to draw incorrect conclusions about political behavior in the 2000 election. For example, as noted above, in Miami-Dade both the ballot image results and Thomsen results show that the proportion of Bush

[9] We omit all precincts created to identify sets of absentee ballots from our analyses, which creates a more conservative test as it reduces the number of cases. The results remain robust for analyses (not shown) run with the inclusion of these precincts.

[10] Because we are concerned with testing the ability of the estimator to recover the individual-level votes and, unlike tests using survey data, we directly create the aggregate-level data from the individual-level data, any errors voters made in terms of recording their intentions on the ballot will be reflected at both the individual and aggregate levels.

[11] The results are robust to disaggregating the combined categories.

[12] Using the individual-level data, among those who voted for Bush, 7.2% voted for the Democratic Senate candidate, Bill Nelson. Among those voting for Gore, 5.9% voted for the Republican candidate for Senate, Bill McCollum. The Thomsen estimates of these proportions are 7.9% and 6.4%, respectively.

voters who split their ticket by voting for the Democratic Senate candidate was larger than the proportion of Gore voters who split their ticket by voting for the Republican Senate candidate. Not only are King's EI estimates far from the mark in absolute terms, they also incorrectly suggest voting for the other major party candidate in the Senate race was more likely among Gore than Bush voters.[13] In other words, while Thomsen's estimator recovered the relative rates of ticket-splitting among Democrats and Republicans, the EI estimator did not. The EI estimates relating to presidential residual votes are similarly misleading. A key result in this election was that those thought most likely to support Democrats were more prone to voting errors. This feature is evident in both the ballot image results (1.54% recorded a residual vote for president and voted for the Senate Democrat while 0.73% recorded a residual vote for president and voted for the Senate Republican) and Thomsen results (2.06% recorded a residual vote for president and voted for the Senate Democrat while 0.76% recorded a residual vote for president and voted for the Senate Republican). The EI results do not follow the same pattern, but rather indicate that more presidential residual voters voted Republican for Senate (0.04% recorded a residual vote for president and voted for the Senate Democrat while 0.05% recorded a residual vote for president and voted for the Senate Republican). In sum, we see that the estimator one uses can clearly influence the accuracy of the conclusions one can draw.[14]

To conduct a more formal analysis of the differences between our ecological inference estimates and the true vote proportions, we measure the index of dissimilarity presented in Thomsen (1987). Thomsen et al. (1991) describe the index of dissimilarity as "the proportion of votes which must be relocated in one sub-table to construct the other sub-table" (447). For example, take the 2×2 tables presented in Fig. 1 and constructed from hypothetical data to represent the actual and estimated vote proportions. In the actual (hypothetical) data (1A), forty-five percent of voters select both the Republican presidential and senatorial candidates (RR), forty-five percent choose both Democratic candidates (DD), and an equal number (five percent each) divide their votes across the two split-ticket options (RD and DR). The estimated results (1B), however, correspond to RR, DD, RD, and DR support rates of forty, fifty, six, and four percent, respectively. The index of dissimilarity between the two matrices is defined as the total sum of absolute deviations divided by two. As such, we first calculate these deviations, which are simply the differences in each cell ($45-40=5$; $5-4=1$; $5-6=-1$; $45-50=-5$). We then take the sum of their absolute value ($5+1+1+5=12$) and divide this figure by two. The last step is taken since the relocation of some values negates the necessity to relocate others (i.e., to alter the actual and estimated proportion of split-ticket voting, a shift of one percent from the RD to DR cell balances those two cells and does not require any reciprocal movement from the DR to RD cell). In this hypothetical scenario, the index of dissimilarity is 6% (12/2).

Returning to Tables 1A and 1B, we find that the apparent impressive performance of the Thomsen estimator suggested by a cursory observation of the tables is confirmed by the implementation of this test. For Miami-Dade County, the index of dissimilarity between the tabulated vote proportions from ballot images and our estimated vote proportions from precinct data is 2.72%. We should note that this index of dissimilarity is lower than those calculated by other studies employing the Thomsen estimator (Hanmer and Traugott, 2004; Thomsen, 2000; Thomsen et al., 1991) and lower than the estimate from King's EI (see Appendix Table B).

While the Thomsen estimator performs best for Miami-Dade (out of all ten counties), it produces impressive results for the other counties as well. For example, in comparing the tabulated vote proportions for Palm Beach (found in Table 2A) with the estimated proportions from the precinct-level data (found in Table 2B), the difference is greater than one percentage point in only one cell (that of the residual votes in the race for president and votes for Nelson), with an overall index of dissimilarity of 3.50%. The test in Palm Beach is particularly interesting given that county's use of the butterfly ballot, which led to an unexpectedly large proportion of votes for Pat Buchanan (Wand et al., 2001). When we separate out the votes for Buchanan from the "others" category, we see that the Thomsen estimator recovers a key feature of the election results in Palm Beach—a disproportionate number of those who registered a vote for Buchanan also voted for Nelson. That is, the Thomsen estimate of the proportion who voted for Buchanan who then selected the Democratic Senate candidate of 72.21% is very close to the actual percentage of 71.99% calculated from the individual-level data.

The results in Sarasota County and Lee County are also quite good. The differences between the tabulated (Table 3A) and estimated (Table 3B) vote proportions for Sarasota County are somewhat larger overall (with an index of dissimilarity of 5.15%), but only in three cases do the cell differences exceed a single percentage point (two percentage points in two of the cases). Even in the case of Lee County (tabulated and estimated proportions found in Tables 4A and 4B, respectively), where the index of dissimilarity (6.59%) is largest, only four comparisons exhibit a difference greater than one percentage point, with only three of these variations near or above two percentage

[13] King's EI estimates the total percentage of those voting for Bush and the Senate Democrat as 0.19% (compared to 3.13% using the actual ballot image data and 3.44% using Thomsen's approach), and the total percentage voting for Gore and the Senate Republican as 0.81% (compared to 3.03% using the ballot image data and 3.25% using Thomsen's approach). Moreover, King's EI estimates the percentage voting for the Senate Democrat among those who voted for Bush as 0.43% (compared to 7.21% using the ballot image data and 7.93% using Thomsen's approach), and the percentage voting for the Senate Republican among those who voted for Gore as 1.59% (compared to 5.93% using the ballot image data and 6.37% using Thomsen's approach).

[14] The results above that compare Thomsen to EI with respect to major party straight- and split-ticket voting reflect a general pattern. Regression-based techniques tend to overstate the degree of straight-ticket voting and thus under-estimate split-ticket voting (see Park, 2008a for a proof confirming this bias). This is the case for EI in every county in our data set, with some substantively large discrepancies. The Thomsen estimator performs better in each case, by a factor as large as 21 and almost always by a factor of 2 or more. We present the rates of major party straight- and split-ticket voting from the actual ballot image data, as well as those generated by the Thomsen and EI estimators, in Appendix Table A.

	R President	D President
R Senate	45%	5%
D Senate	5%	45%

1A - Tabulated (Hypothetical) Results

	R President	D President
R Senate	40%	4%
D Senate	6%	50%

1B - Estimated (Hypothetical) Results

Fig. 1. Illustration of index of dissimilarity calculation via hypothetical data.

Table 2A
Tabulated vote proportions from ballot images, Palm Beach 2000.

		U.S. Senate race				
		Republican	Democrat	Others	Resid. votes	Total
President	Bush	26.27%	3.90%	0.55%	1.16%	130,686
	Gore	4.32%	50.69%	1.37%	2.62%	241,802
	Others	0.57%	1.26%	0.33%	0.20%	9673
	Resid. votes	1.34%	3.64%	0.26%	1.50%	27,641
	Total	133,195	243,816	10,300	22,491	409,802

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 3.50% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

Table 2B
Estimated vote proportions using Thomsen estimator from precinct-level data ($N = 506$), Palm Beach 2000.

		U.S. Senate race			
		Republican	Democrat	Others	Resid. votes
President	Bush	26.77%	3.06%	1.00%	1.06%
	Gore	3.80%	50.51%	1.23%	3.47%
	Others	1.05%	1.06%	0.10%	0.16%
	Resid. votes	0.89%	4.87%	0.18%	0.80%

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 3.50% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

Table 3A
Tabulated vote proportions from ballot images, Sarasota 2000.

		U.S. Senate race				
		Republican	Democrat	Others	Resid. votes	Total
President	Bush	42.28%	5.21%	0.88%	1.44%	69,836
	Gore	4.75%	37.71%	1.27%	1.73%	63,742
	Others	1.02%	1.30%	0.62%	0.21%	4411
	Resid. votes	0.44%	0.42%	0.06%	0.64%	2201
	Total	67,968	62,593	3969	5660	140,190

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 5.15% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

Table 3B
Estimated vote proportions using Thomsen estimator from precinct-level data ($N = 141$), Sarasota 2000.

		U.S. Senate race			
		Republican	Democrat	Others	Resid. votes
President	Bush	44.17%	4.05%	0.55%	1.05%
	Gore	2.46%	38.29%	2.01%	2.70%
	Others	1.23%	1.53%	0.22%	0.17%
	Resid. votes	0.63%	0.78%	0.05%	0.12%

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 5.15% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

Table 4A
Tabulated vote proportions from ballot images, Lee 2000.

		U.S. Senate race				
		Republican	Democrat	Others	Resid. votes	Total
President	Bush	48.23%	4.90%	0.90%	1.10%	92,604
	Gore	6.35%	31.14%	1.28%	1.21%	67,155
	Others	0.97%	0.92%	0.50%	0.12%	4224
	Resid. votes	1.02%	0.60%	0.10%	0.66%	3994
	Total	95,041	63,080	4660	5196	167,977

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 6.59% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

Table 4B
Estimated vote proportions using Thomsen estimator from precinct-level data ($N = 150$), Lee 2000.

		U.S. Senate race			
		Republican	Democrat	Others	Resid. votes
President	Bush	50.94%	2.94%	0.44%	0.82%
	Gore	3.52%	32.28%	2.13%	2.05%
	Others	1.20%	1.10%	0.13%	0.09%
	Resid. votes	0.93%	1.24%	0.08%	0.14%

Note: Table A presents the actual vote proportions tabulated from the ballot images, while Table B presents the Thomsen estimates of the vote proportions by using precinct-level data aggregated from the ballot images. Index of dissimilarity = 6.59% (measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two).
Source: 2000 Florida Ballots Project.

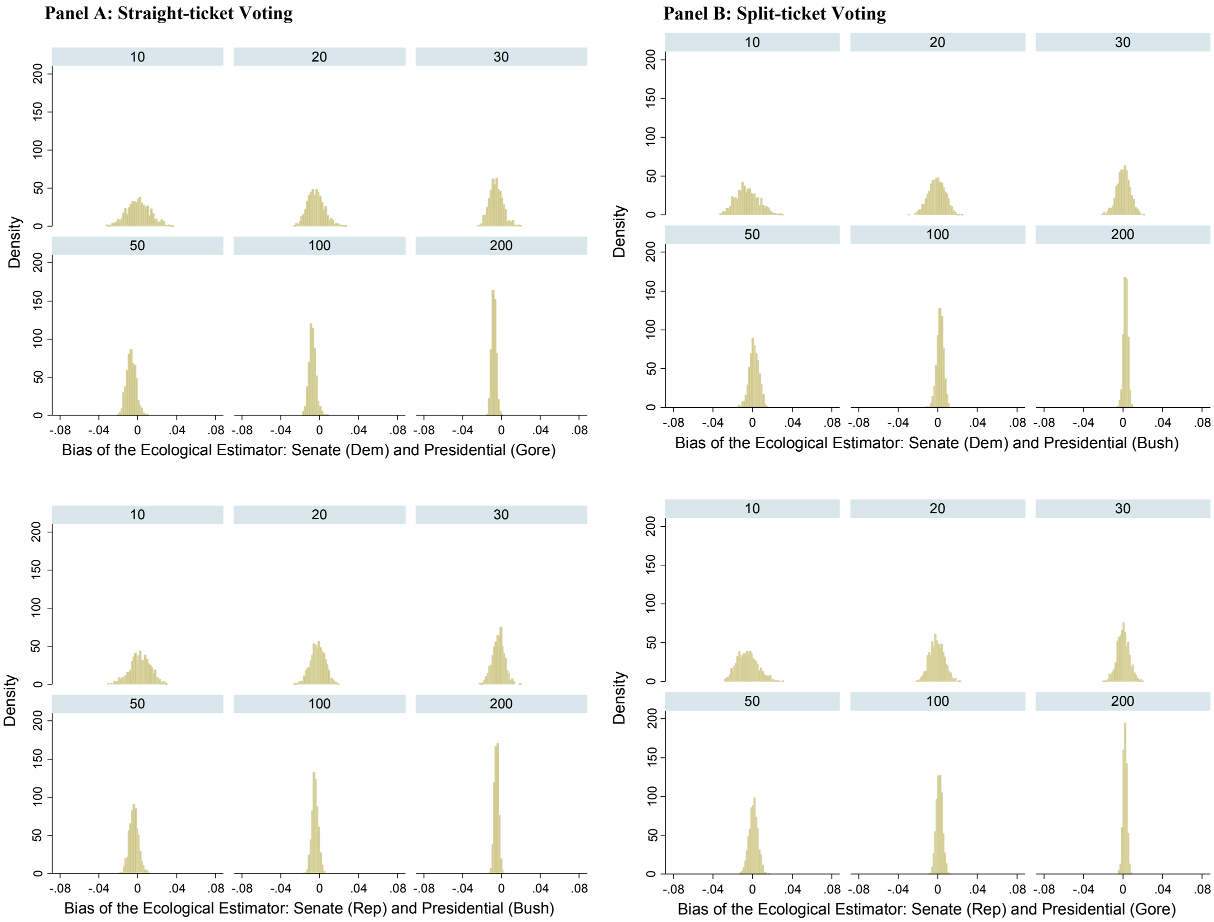

Fig. 2. Bias of the Thomsen estimates by sample size for Miami-Dade County. Note: Figures show the bias of the estimates in voting for the specified presidential and senatorial candidates. Bias is defined as the difference between the ecological estimate of the voting proportion and the actual ballot tabulation (i.e. a bias of zero means that the estimated and actual proportions are the same). Source: 2000 Florida Ballots Project.

points. This minimal variation between the estimated and actual vote proportions across all counties, combined with index of dissimilarities that compare favorably with those calculated by other studies (Hanmer and Traugott, 2004; Thomsen, 2000; Thomsen et al., 1991), demonstrates the ability of the Thomsen estimator to generate estimates close to the individual-level estimates when faced with significant diversity (both racial/ethnic and partisan).

4.2. Thomsen estimator and small samples

The substantial similarity between the actual vote proportions and our estimates for these counties provides convincing evidence of the robustness of the Thomsen estimator to significant heterogeneity within and across aggregate units. The counties analyzed here are also diverse in terms of their size. Though the Thomsen estimator tended to perform better in the counties with a large number of precincts, it also performed admirably in the counties with a small number of precincts. For example, though the estimates from Highlands County are based on only 18 precincts, the index of dissimilarity was just 5.78% (for more see Web Appendix Tables 1A–6B). We now turn to a stricter test of the performance of the Thomsen estimator as the number of aggregate units decreases.

To perform this more rigorous test, we developed a resampling procedure using the individual-level ballot data from Florida. First, for each county within a specified unit of precincts (10, 20, 30, 50, 100, 200), we drew individual ballots from a random sample of precincts. Second, we calculated the relevant vote proportions and treated these results as the “truth”. Third, we aggregated the individual ballots by precinct. Fourth, we estimated the relevant vote proportions based on the aggregate units generated from the individual ballots. Fifth, for each cell, we calculated the bias in the aggregate estimate, defined as the difference between the proportions calculated from the individual ballot data and estimated proportion from the aggregated precinct data. Finally, we repeated this process one thousand times for each sample size.

Because the procedure calculates the bias for each individual cell, we do not have space to present all of these results for a single county, let alone the four counties discussed above. Instead, we focus on the two counties with the lowest

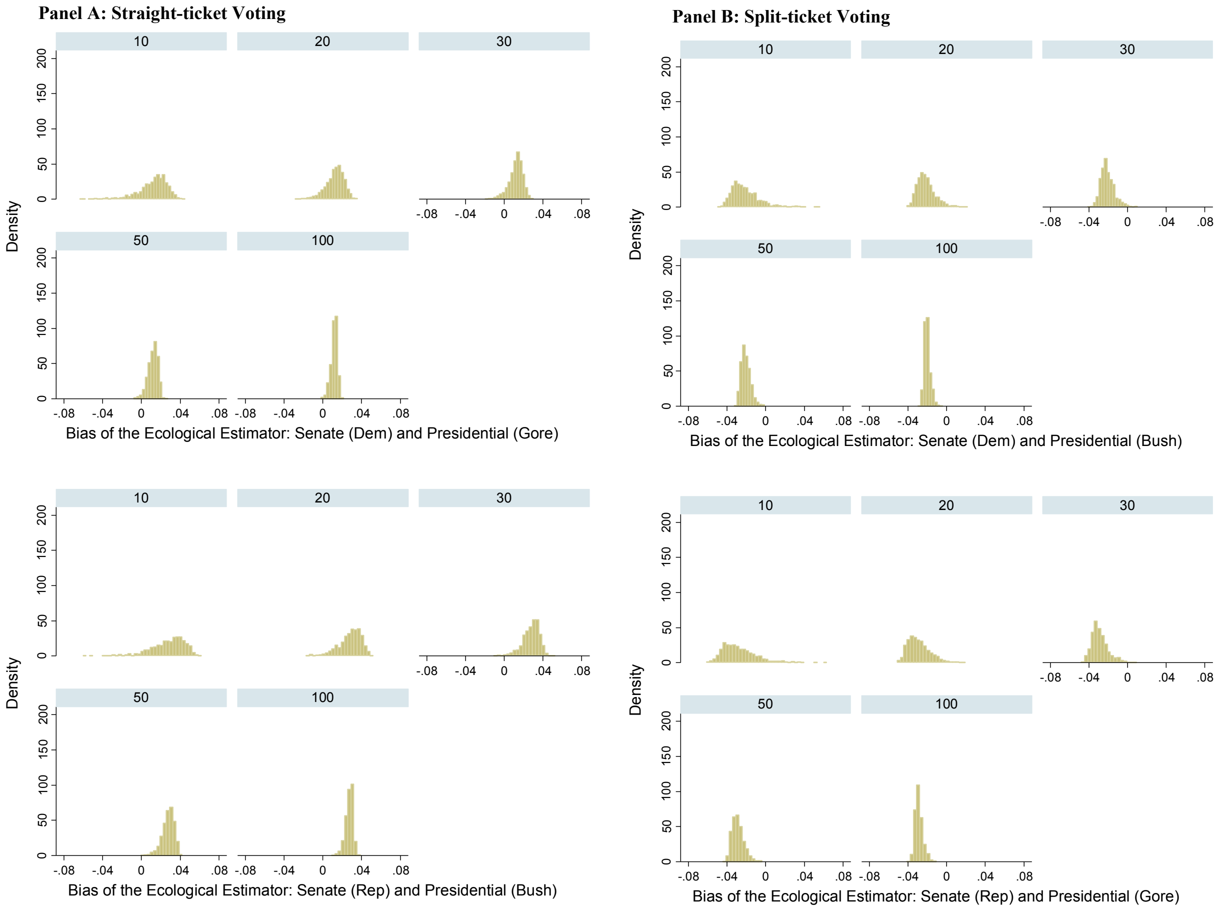

Fig. 3. Bias of the Thomsen estimates by sample size for Lee County. Note: Figures show the bias of the estimates in voting for the specified presidential and senatorial candidates. Bias is defined as the difference between the ecological estimate of the voting proportion and the actual ballot tabulation (i.e. a bias of zero means that the estimated and actual proportions are the same). Source: 2000 Florida Ballots Project.

and highest indices of dissimilarity (Miami-Dade and Lee,[15] respectively), and only discuss the instances of major party straight and split-ticket voting (i.e. Bush and McCollum, Gore and Nelson, Bush and Nelson, Gore and McCollum).[16]

The estimates are presented in Fig. 2 (Miami-Dade) and Fig. 3 (Lee), with the straight-ticket estimates shown in Panel A and the split-ticket estimates in Panel B. In Miami-Dade, across all four patterns of behavior the bias is quite small. Even at just fifty observations the errors never exceed an absolute value of 0.02. Though the variance in this distribution decreases as the sample size increases, even at thirty observations the bias remains close to the absolute value of 0.02. For the estimates of split-ticket voting, Gore to McCollum and Bush to Nelson, the estimate of the bias is quite close to zero, even at a sample size of just twenty precincts (see Panel B of Fig. 2). While the estimates for Lee County are more biased than those for Miami-Dade, overall the bias tends to be small here as well. With as few as thirty observations, across almost all of the runs the bias is less than 0.04 in absolute value, and the estimator does equally well in retrieving all four combinations of straight and split-ticket voting. Thus, even with the county that proves most difficult for the Thomsen estimator, accurate estimates of voting behavior can be recovered from as little as thirty aggregate units.[17]

As a final and even more conservative test of the Thomsen estimator, we combine the precinct data from all ten counties to create a hypothetical "state" comprised of 2915 precincts that exhibits greater variation than any single county in the analysis. We perform the same resampling procedure used for the individual counties, and present the results in Fig. 4 for straight and split-ticket voting for presidential and senatorial candidates (results are similar across all combinations of cells).[18] The low levels of bias in

[15] Since Lee County has fewer than 200 precincts we vary the sample size only up to 100 precincts.

[16] While we focus on these comparisons, the results are similar across all cells and across all counties. Estimates of the biases not presented here are available from the authors upon request.

[17] We also evaluated how the index of dissimilarity is affected by changes in the number of aggregate units. Consistent with the general patterns discussed earlier, the results indicate that the Thomsen estimator performs well even when the number of aggregate units drops to thirty. These results are available upon request.

[18] The index of dissimilarity between the actual and estimated vote proportions for the hypothetical state is 2.42%.

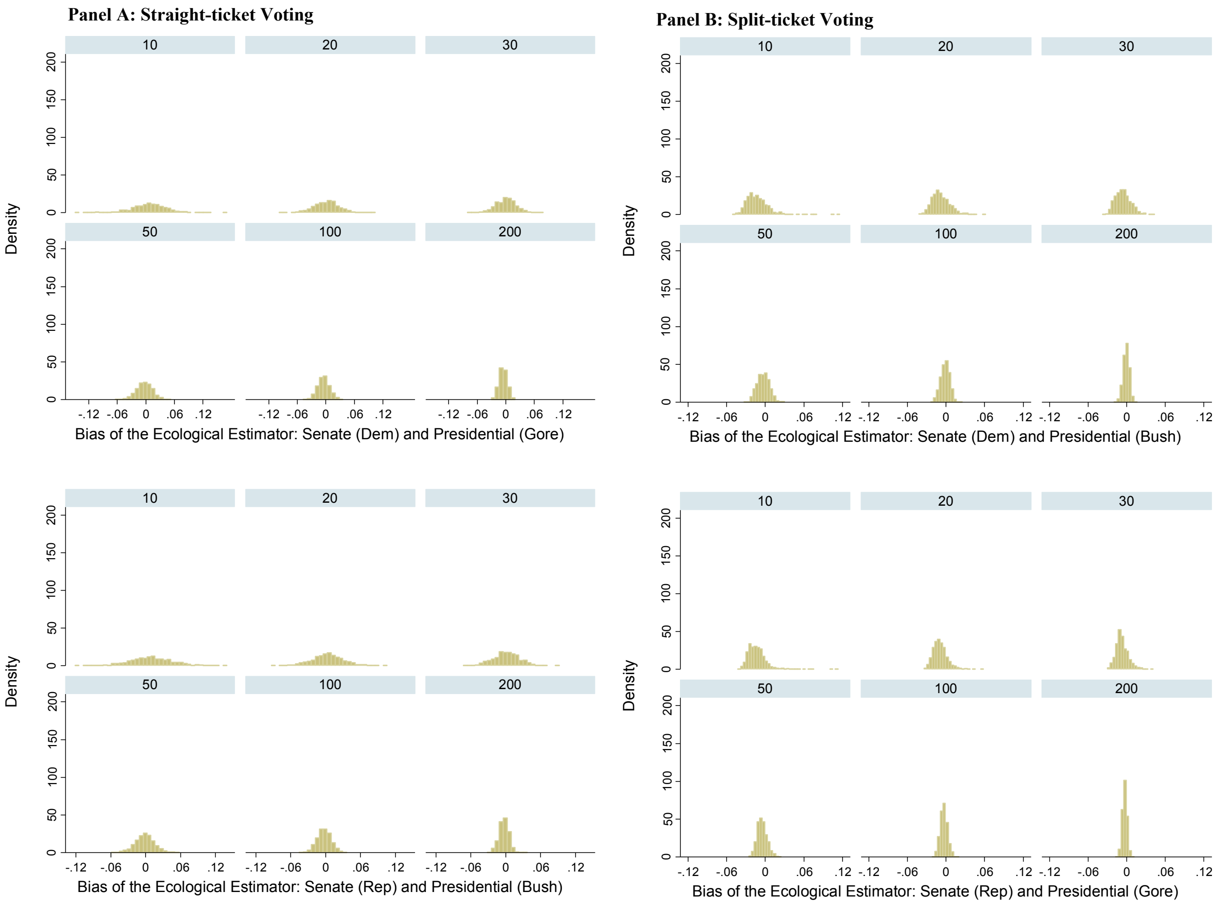

Fig. 4. Bias of the Thomsen estimates by sample size for hypothetical state. Note: Figures show the bias of the estimates in voting for the specified presidential and senatorial candidates. Bias is defined as the difference between the ecological estimate of the voting proportion and the actual ballot tabulation (i.e. a bias of zero means that the estimated and actual proportions are the same). Source: 2000 Florida Ballots Project.

the estimates are remarkable; while draws of ten or twenty precincts produce noticeable tails in the distribution of the errors, the estimator recovers the “state's” voting patterns with limited bias using as few as thirty or fifty precincts. These results remain robust even if we remove Miami-Dade Country, demonstrating that its large number of precincts and the strong performance of the estimator when recovering its individual-level vote proportions (in comparison to the other counties) do not drive the estimator's performance regarding the hypothetical state.

5. Conclusion

Problems of ecological inference have long been of interest to scholars, who have developed a number of approaches to address them. One method, the Thomsen estimator, is particularly appealing for scholars of electoral dynamics given its theoretical foundation. Importantly, the Thomsen estimator recovers the individual-level behavior quite well with regards to voter transition rates across elections and across races within an election, generating closer estimates to the individual results than those found in previous analyses. This conclusion holds when the population under investigation is diverse on both demographic and partisan measures across and within the aggregate-level units. The estimator also performs well even when the number of aggregate units is quite small, demonstrating the robustness of the Thomsen estimator to changes in sample size.

The robustness of the Thomsen estimator to both diverse and small samples suggests significant confidence in its application to a number of questions in the study of elections. In addition to general interest in straight- and split-ticket voting, there are a variety of applications in the realm of election reform; the study of changes in voting machines, early voting, absentee voting, and changes in ballot format (e.g., straight-party device) could all benefit from the implementation of this statistical technique. The evidence presented here suggests that scholars can have significant confidence in applying the estimator to problems of ecological inference, even when the nature of the data is less than ideal.

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Appendix A. Supplementary material

Supplementary data related to this article can be found at http://dx.doi.org/10.1016/j.electstud.2014.08.006.

Appendix Table A
Total percentage of major party straight-ticket voters using King's EI, Thomsen's estimator, and individual ballot images in Florida Counties, 2000.

County	King's EI	Thomsen estimator	Ballot images	Difference: King − Ballot images	Difference: Thomsen − Ballot images
Broward	90.95%	84.68%	83.91%	7.04%	0.77%
Highlands	86.94%	83.75%	80.37%	6.57%	3.38%
Hillsborough	85.96%	78.17%	79.76%	6.20%	−1.59%
Lee	84.26%	83.22%	79.37%	4.89%	3.85%
Marion	86.40%	82.64%	79.93%	6.47%	2.71%
Miami-Dade	84.96%	76.45%	77.78%	7.18%	−1.33%
Palm Beach	83.79%	77.28%	76.96%	6.83%	0.32%
Pasco	83.89%	80.95%	77.96%	5.93%	2.99%
Pinellas	87.07%	82.49%	79.42%	7.65%	3.07%
Sarasota	87.23%	82.46%	79.99%	7.24%	2.47%

Note: EI analysis run in R using the ecological inference code integrated into Zelig (Imai et al., 2007a,b), which implements models using a nonlinear least squares approximation (Wittenberg et al., 2007). In contrast to earlier ecological inference approaches that are Bayesian in nature (see for example Rosen et al., 2001), this strategy implements a frequentist approximation of these Bayesian models. As such, it is not Bayesian by design and does not require priors or starting values to be specified.
Source: 2000 Florida Ballots Project.

Appendix B

Appendix Table B
Index of dissimilarity comparisons for Thomsen's estimator and King's EI for voting patterns in Florida Counties, 2000.

	Thomsen estimator	King's EI
Broward	3.04	8.38
Highlands	5.78	9.36
Hillsborough	3.95	8.71
Lee	6.59	7.58
Marion	5.25	8.15
Miami-Dade	2.72	10.78
Palm Beach	3.50	9.57
Pasco	5.95	7.59
Pinellas	5.67	9.59
Sarasota	5.15	9.44

Note: Index of dissimilarity is the measurement of the difference between the two transition matrices, defined as the total sum of absolute deviations divided by two. EI analysis run in R using the ecological inference code integrated into Zelig (Imai et al., 2007a,b), which implements models using a nonlinear least squares approximation (Wittenberg et al., 2007). In contrast to earlier ecological inference approaches that are Bayesian in nature (see for example Rosen et al., 2001), this strategy implements a frequentist approximation of these Bayesian models. As such, it is not Bayesian by design and does not require priors or starting values to be specified.
Source: 2000 Florida Ballots Project.

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